Roll No. ....

72461

M. Sc. Mathematics 1st Semester
CBCS Current Scheme w. e. f. 2016-17
Examination – December, 2024

ABSTRACT ALGEBRA

Paper : 16MAT21C1

Time : Three Hours] *[Maximum Marks : 80*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt *five* questions in all, selecting *one* question from each Section. Question No. 9 (Section - V) is *compulsory*. All questions carry equal marks.

SECTION – I

1. (a) State and prove sylow first theorem. 8

   (b) Show that a sylow p-subgroup of a finite group is unique iff it is normal subgroup. 8

2. (a) Show that a group of order 108 is not simple. 8

(b) Let P be a sylow p-subgroup of G and let $x \in N(P)$ be an element such that $O(x) = p^r$. Then, show that $x \in P$. 8

SECTION – II

3. (a) If a group G has a subnormal series with solvable factors, then show that G is solvable. 8

(b) State and prove Zassenhaus Lemma. 8

4. (a) Prove that a simple group is solvable iff it is cyclic group of prime order. Further, show that if a group has a composition series and H be a proper normal subgroup of G, then G has a composition series containing H. 8

(b) Let G be a non-trivial nilpotent group and H be a non-trivial normal subgroup of G, then prove that $H \cap Z(G) \neq \{e\}$. 8

SECTION – III

5. (a) State and prove fundamental theorem on finitely generated modules over Euclidean rings. 10

(b) Suppose $1 \in R$, then prove that M is a cyclic R-module iff $M \cong R/I$, where I is a left ideal of R. 6

6. (a) Let N be a finitely generated free module over a commutative ring R. Then, prove that all its basis are finite and have same number of elements. 8

(b) For any module M over a commutative integral domain R, prove that the quotient module M/M_i is a torsion free module. 8

SECTION – IV

7. (a) Prove that an R-module M is Artinian iff every quotient module of M is finitely co-generated. 8

(b) State and prove Hilbert basis theorem. 8

8. (a) Prove that in a left noetherian ring every ideal contains a finite product of prime ideals. 8

(b) State and prove Maschk theorem. 8

SECTION – V

9. (a) Give example of a ring which is Noetherian but not Artinian. $8 \times 2 = 16$

(b) Differentiate nil and nilpotent ideals.

(c) Show that Q is not a free Z-module.

(d) Prove that if R is an integral domain, then Torm is a submodule of the R-module M.

(e) Let H be an abelian normal subgroup of a group G such that $O(G/H) = 91$. Prove that G is solvable.

(f) Write down two composition series for z_8.

(g) Find all non-isomorphic groups of order 6.

(h) Show that a group of order 56 is not simple.

Roll No. ....

72462

M. Sc. Mathematics 1st Semester
CBCS Current Scheme w. e. f. 2016-17
Examination – December, 2024

MATHEMATICAL ANALYSIS

Paper : 16MAT21C2

Time : Three Hours] *[Maximum Marks : 80*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt ***five*** questions in all, selecting ***one*** question from each Section. Question No. 9 (Section - V) is ***compulsory***. All questions carry equal marks.

SECTION – I

1. (a) For any two partitions P_1 and P_2 on $[a, b]$, let f be a bounded real valued function defined on $[a, b]$ and α is monotonically increasing function defined on $[a, b]$, then prove that $L(P_1, f, \alpha) \leq U(P_2, f, \alpha)$. 8

P. T. O.

(b) Let α is monotonically increasing on $[a, b]$, then prove that $f \in R(\alpha)$ iff for any $\varepsilon > 0$, there exists a partion P of $[a, b]$ such that $U(P, f, \alpha) - L(P, f, \alpha) < \varepsilon$. 8

2. (a) Let $f_1, f_2 \in R(\alpha)$ on $[a,b]$, then prove that $f_1 + f_2 \in R(\alpha)$ and $\int_a^b (f_1 + f_2)\, d\alpha = \int_a^b f_1 d\alpha + \int_a^b f_2 d\alpha$. 8

(b) State and prove first mean value theorem for Riemann-Stieltje's integral. 8

SECTION – II

3. (a) State and prove Cauchy criterion for uniform convergence of a series of functions. 8

(b) State and prove Dirichlet's test for uniform convergence. 8

4. (a) Show that the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n + x^2}$ is uniformly convergent but not absolutely for all real values of x. 8

(b) Let α be monotonically increasing on $[a,b]$. Suppose that each term of the sequence $\{f_n\}$ is a real valued function such that $f_n \in R(\alpha)$ on $[a, b]$ for all n and suppose $f_n \to f$ uniformly on $[a, b]$. Then, prove that $f \in R(\alpha)$ on $[a,b]$ and $\int_a^b f d\alpha = \lim_{n \to \infty} \int_a^b f_n d\alpha$. 8

SECTION – III

5. (a) Let $\sum_{n=0}^{\infty} a_n x^n$ be a power series with finite radius of convergence R and let $f(x) = \Sigma a_n x^n ; |x| < R$. If the series $\Sigma a_n x^n$ convergence at end point $x = R$, then prove that $\lim_{x \to R^-} f(x) = \Sigma a_n R^n$. 8

(b) Let E be an open set in R^n and f maps E in R^m and $x \in E$. Suppose $h \in R^n$ is small enough such that $x + h \in E$. Then, prove that f has a unique derivative. 8

6. (a) State and prove Young's theorem. 8

(b) State and prove Tauber's theorem. 8

SECTION – IV

7. (a) If $f(x, y) = \sqrt{|xy|}$, prove that Taylor's expansion about the point (x, x) is not valid in any domain which includes the origin. 8

(b) State and prove implicit function theorem. 8

8. (a) If $x = r\cos\theta, y = r\sin\theta$, then prove that : 8

$$(x^2 - y^2)\left(\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2}\right) + 4xy\frac{\partial^2 u}{\partial x \partial y} = r^2\frac{\partial^2 u}{\partial r^2} - r\frac{\partial u}{\partial r} - \frac{\partial^2 u}{\partial \partial^2}$$

Where u is any twice differentiable functions of x and y.

P. T. O.

(b) If $u_1, u_2, \ldots, u_n$ be n functions of n variables $x_1, x_2, \ldots, x_n$, say, $u_m = f_m(x_1, x_2, \ldots, x_m)$, $(m = 1, 2, \ldots, n)$ and if $\frac{\partial(u_1, u_2, \ldots, u_n)}{\partial(x_1, x_2, \ldots, x_n)} = 0$, then if all differential co-efficients concerned are continuous, there exists a functional relation connecting some on all of the variables $u_1, u_2, \ldots, u_n$ which is independent of $x_1, x_2, \ldots, x_n$. 8

SECTION – V

9. (a) Define rectifiable curves. $8 \times 2 = 16$

(b) State fundamental theorem of calculus.

(c) State Weierstrass m-test.

(d) Define point wise convergence.

(e) Define radius of convergence of a power series.

(f) What is chain rule ?

(g) Define explicit and implicit functions.

(h) What is stationary values of implicit functions ?

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72464

M. Sc. Mathematics 1st Semester
CBCS Current Scheme w.e.f. 2016-17
Examination – December, 2024

COMPLEX ANALYSIS

Paper : 16MAT21C4

Time : Three hours] *[Maximum Marks : 80*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt ***five*** questions in all, selecting ***one*** question from each Section. Question No. 9 (Section-V) is ***compulsory***. All questions carry equal marks.

SECTION – I

1. (a) State and prove the sufficient condition for the function $f(z)$ to be analytic. 8

   (b) Prove that $u(x, y) = e^{-x}(x \sin y - y \cos y)$ is harmonic and find the conjugate harmonic function $v(x, y)$, such that $f(z) = u + iv$ is analytic. 8

P. T. O.

2. (a) Show that the sum function $f(z)$ of the power series $\sum_{n=0}^{\infty} a_n z^n$ represents an analytic function inside its circle of convergence. 8

(b) Examine the behaviour of the power series $\sum_{n=2}^{\infty} \frac{z^n}{n(\log n)^2}$ on the circle of convergence. 8

SECTION – II

3. (a) State and prove Cauchy's elementary theorem. 6

(b) If $f(z)$ is analytic inside and on a closed contour C, it possesses derivatives of all orders which are all analytic inside C. The show that nth derivative $f^n(z_0)$ at any point z_0 inside C being given by the formula $f^n(z_0) = \frac{\lfloor n}{2\pi i} \int_C \frac{f(z)dz}{(z-z_0)^{n+1}}$. 10

4. (a) State and prove Poisson's integral formula. 8

(b) Show that a function which is analytic in all finite regions of the complex plane and bounded, is identically equal to a constant. 8

SECTION – III

5. (a) Show that $e^{\frac{c}{2}\left(z-\frac{1}{z}\right)} = \sum_{n=-\infty}^{\infty} a_n z^n$, where

$$a_n = \frac{1}{2\pi} \int_0^{2\pi} \cos(n\theta - c\sin\theta)d\theta.$$ 8

(b) Describe different types of singularities in detail. 8

6. (a) State and prove Weierstrass Theorem. 8

(b) Let f(z) be analytic inside and on a simple closed contour C except for a finite number of poles inside C and let $f(z) \neq 0$ on C, then $\frac{1}{2\pi i}\int_C \frac{f'(z)}{f(z)}dz = N - P$, where N and P are respectively the total number of zeros and number of poles of $f(z)$ inside C, a zero (pole) of order m being counted m times. 8

SECTION – IV

7. (a) State and prove Cauchy Residue Theorem. 8

(b) Prove that $\int_0^{2\pi} e^{-\cos\theta}\cos(\sin\theta + n\theta)d\theta = \frac{2\pi(-1)^n}{n!}$, where n is a +ve integer. 8

8. (a) If the mapping $w = f(z)$ is conformal and there exist a pair of continuously differentiable relations $u = u(x, y)$, $v = v(x, y)$ then show that $f(z)$ is an analytic function of z. 8

(b) Show that a family Φ in $H(G)$ is normal iff Φ is locally bounded. 8

P. T. O.

SECTION – V

9. (a) Show that an analytic function with constant modulus is constant.

(b) State Cauchy's Inequality.

(c) Define Circle of Convergence.

(d) Using Cauchy's integral formula, prove that

$$\int_C \frac{e^z}{z-\pi i}\,dz = -2\pi i, \text{ where C is the circle } |z| = 4.$$

(e) State Schwarz's lemma.

(f) Using Rouche's theorem, find the number of roots of $z^8 - 4z^5 + z^2 - 1 = 0$ lying inside $|z| = 1$.

(g) Find the residue of $\dfrac{z^4}{z^2 + a^2}$ at $z = -ia$.

(h) Define Conformal Mapping. $8 \times 2 = 16$

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86091

Master of Science (Mathematics) 1st Sem. Examination – December, 2024

ABSTRACT ALGEBRA

Paper : 24MAT201DS01

Time : Three Hours] *[Maximum Marks : 70*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt ***five*** questions in all, selecting ***one*** question from each Section. Question No. **1** is ***compulsory***. All questions carry equal marks.

1. *Compulsory* Question :

(i) Show that a group of order 28 is not simple. 2

(ii) Prove that S_4 is solvable. 2

(iii) How can we find order of a double coset ? 1

(iv) Prove that centre of a nilpotent group G is always non-trivial. 2

(v) Give equivalent conditions for a Noetherian ring. 2

(vi) Describe opposite rings. 1

(vii) State fundamental structure theorem of finitely generated modules over principal ideal domain. 2

(viii) Prove that the kernel of a homomorphism is a submodule. 2

SECTION – I

2. (a) Let G be a finite group of order p^n, where p is a prime. Show that G has subgroups of order 1, p, p^2, ... p^n. 7

(b) Prove that any two Sylow p-subgroups of a finite group G are conjugates in G. 7

3. (a) State and prove Sylow's third Theorem. 7

(b) Let G be a finite group such that, $o(G) = p^n$ where p is a prime. Prove that any subgroup of order p^{n-1} is a normal subgroup of G. 7

SECTION – II

4. (a) Prove that a group G is solvable iff $G^{(n)} = \langle e \rangle$ for some $n \geq 0$. 7

(b) Let G be a group and H be a normal subgroup of G. Then, if H and G/H both are solvable then prove that G is also a solvable group. 7

5. (a) If H is a central subgroup of G. Also, $H \trianglelefteq G$, both H and G/H are nilpotent subgroups of G. Then, prove that G must also nilpotent. 7

(b) State and prove Schreier's Refinement Theorem. 7

SECTION – III

6. (a) State and prove Schur's lemma. 7

(b) Prove that Q is not a free Z-module. 7

7. (a) Prove that let R be a ring with unity and M is a free R module with basis $(e_1, e_2, \ldots\ldots, e_n)$, then $M \cong R^n$. 7

(b) Prove that if N is a finitely generated free module over a commutative ring R, then all its bases are finite. 7

SECTION – IV

8. (a) Prove that direct sum of Noetherian modules is Noetherian. 7

(b) Let M be a left R-module and N be a submodule of M, then prove that M is Artinian iff N and M/N both are Artinian. 7

P. T. O.

9. (a) State and prove Wedderburn Artin theorem. 7

(b) Let R be a left Artinian ring with unity having no nonzero nilpotent ideals, then prove that every nonzero left ideal of R contains nonzero idempotents. 7

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86092

Master of Science (Mathematics)
1st Semester
Examination – December, 2024

MATHEMATICAL ANALYSIS

Paper : 24MAT201DS02

Time : Three Hours] *[Maximum Marks : 70*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt ***five*** questions in all, selecting ***one*** question from each Section. Question No. **1** is ***compulsory***. All questions carry equal marks.

1. ***Compulsory question :*** $2 \times 7 = 14$

(a) What do you mean by refinement of a partition and common refinement of partitions ?

(b) Evaluate $\int_0^2 x^2 dx^2$.

(c) If f is continuous function on [0, 1] and if $\int_0^1 x^n f(x)\,dx = 0$, for $n = 0, 1, 2, \ldots\ldots$ then $f(x) = 0$ on [0, 1].

(d) State Cauchy's criterion for uniform convergence for series of functions.

(e) Find the convergence for the power series $x + \frac{x^2}{2^2} + 2!\frac{x^3}{3^3} + \frac{3!}{4^4}x^4 + \ldots$

(f) Define Jacobian.

(g) State Abel's theorem.

SECTION – I

2. (a) $f \in R(\alpha)$ on $[a, b]$ iff for every $\varepsilon > 0$ there exists a partition p s.it. $U(P, f, \alpha) - L(P, f, \alpha) < \varepsilon$ 7

(b) If $f \in R(\alpha)$ and $c > 0$, then $f \in R(c\alpha)$ and $\int_a^b f\, d(c\alpha) = c\int_a^b f\, d\alpha$. 7

3. (a) State and prove fundamental theorem of calculus. 7

(b) Let f be a constant function on [a, b] defined as $f(x) = k$ and α be a monotonically increasing function of [a, b]. Then prove that $\int_a^b f\, d\alpha$ exists and $\int_a^b f\, d\alpha = k[\alpha(b) - \alpha(a)]$. 7

SECTION – II

4. (a) Differentiate between pointwise convergence and uniform convergence for sequence of functions. Also show that the sequence of functions $\{f_n\}$, where $f_n(x) = \frac{1}{x+n}$ is uniformly convergent in any interval $[0, b]$, $b > 0$. 7

(b) Show that the sequence of functions $\{f_n\}$, where $f_n(x) = \frac{x}{1+nx^2}$, $x \in R$ is uniformly convergent on any interval containing zero by M_n-test. 7

5. (a) State and prove Weirstrass M-test for uniform convergence. 7

(b) If a sequence of continuous functions $\{f_n\}$ defined on $[a, b]$ is monotonically increasing and converges pointwise to a continuous function f, then the convergence is uniform on $[a, b]$. 7

SECTION – III

6. (a) State and prove Tauber's theorem. 7

(b) If a power series $\sum_{n=0}^{\infty} a_n x^n$ converges at the end point R of the interval of convergence $(-R, R)$, then it is uniformly converges in the interval $[0, R]$. 7

7. (a) Give an example to show that the partial derivatives need not be always differential coefficient. 7

(b) With the help of an example, show that the conditions of Young's theorem are sufficient but not necessary. 7

SECTION – IV

8. (a) State and prove Taylor's theorem. 7

(b) Transform the expression :

$$\left(x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}\right)^2+(a^2-x^2-y^2)\left\{\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2\right\}$$

by the substitution $x = r\cos\theta,\ y = r\sin\theta$. 7

9. (a) Show that $f(x,y)=2x^4-3x^2y+y^2$ has neither maximum nor minimum at (0, 0). 7

(b) If $u = x+2y+z,\ v = x-2y+3z$ and $w = 2xy - xz + 4yz - 2z^2$, prove that $\dfrac{\partial(u,v,w)}{\partial(x,y,z)} = 0$ and find a relation between u, v and w. 7

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Master of Science Mathematics 1st Semester Examination – December, 2024

COMPLEX ANALYSIS

Paper : 24MAT201DS03

Time : Three hours] *[Maximum Marks : 70*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt ***five*** questions in all, selecting ***one*** question from each Section. Question No. 9 (Section-V) is ***compulsory***. All questions carry equal marks.

SECTION – I

1. (a) For the function $f(z)=\begin{cases}(\bar{z})^2/z, & z\neq 0\\ 0, & z=0\end{cases}$, $z=x+iy$, show that $f(z)$ is not differentiable at the origin, although C-R equations are satisfied at that point. 7

(b) State and prove sufficient condition for $f(z)$ to be analytic. 7

P. T. O.

2. (a) Prove that $u(x, y) = e^{-x}(x \sin y - y \cos y)$ is harmonic and find the conjugate harmonic function $v(x, y)$ such that $f(z) = u + iv$ is analytic. 7

(b) Discuss the branches, branch cut and branch point for the function $w = z^{1/2}$. 7

SECTION – II

3. (a) If $f(z)$ is analytic in a ring shaped region bounded by two closed contours C_1 and C_2 and Z_0 is a point in the region between C_1 and C_2, then show that $f(z_0) = \frac{1}{2\pi i}\int_{C_2}\frac{f(z)}{z - z_0}dz - \frac{1}{2\pi i}\int_{C_1}\frac{f(z)}{z - z_0}dz$, where C_2 is the outer contour. 7

(b) State and prove Poisson's integral formula. 7

4. (a) Show that the bounded entire functions are the constant functions. 6

(b) State and prove Taylor's theorem. 8

SECTION – III

5. (a) Expand $f(z) = \frac{1}{z(z-3)}$ in a Laurent's series valid for $1 < |z + 1| < 4$. 7

(b) If an analytic function $f(z)$ has a pole of order m at $z = a$, then show that $\frac{1}{f(z)}$ has a zero of order m at $z = a$ and conversely. 7

6. (a) Let $f(z)$ be analytic in a domain D defined by $|z| < R$ and let $|f(z)| \leq M$ for all z in D and $f(0) = 0$, then $|f(z)| \leq \frac{M}{R}|z|$. Also, if the equality holds for any one z, then $f(z) = \frac{M}{R} z e^{i\alpha}$ where α is real constant. 7

(b) State and prove Inverse Function theorem. 7

SECTION – IV

7. (a) Evaluate $\int_0^{\pi} \frac{\sin^4 \theta}{a + b\cos\theta} d\theta$ where $a > b > 0$. 7

(b) By method of contour integration, prove that : 7

$$\int_{-\infty}^{\infty} \frac{x^2 - x + 2}{x^4 + 10x^2 + 9} . dx = \frac{5\pi}{12}$$

8. (a) Prove that at each point z of a domain where $f(z)$ is analytic and $f'(z_0) = 0$, z_0 being an interior point, the mapping $w = f(z)$ is conformal. 7

(b) State and prove Hurwitz's Theorem. 7

SECTION – V

9\. (a) Define single valued function and multivalued function. $2 \times 7 = 14$

(b) For what value of z, the function w defined by $z = e^{-v}(\cos u + i \sin u)$ ceases to be analytic.

(c) State Cauchy elementary theorem.

(d) Using Cauchy's integral formula, prove that :

$$\int_C \frac{e^z}{z - \pi i} dz = -2\pi i,$$

where C is the circle $|z| = 4$.

(e) Show that e^{-1/z^2} has no singularity.

(f) State Jordan lemma.

(g) Find all points where the mapping $f(z) = \sin z$ is conformal.

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Master of Science Mathematics 1st Sem. Examination – December, 2024

MATHEMATICAL STATISTICS

Paper : 24MAT201DS04

Time : Three Hours] *[Maximum Marks : 70*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt *five* questions in all, selecting ***one*** question from each Section. Question No. 1 is ***compulsory***. All questions carry equal marks.

1. (a) Give statistical definition of probability. 1½

(b) What are 'pairwise' and 'mutually' independent events ? 2

(c) Define a random variable. Give an example. 2

(d) For a random variable X, show that

$$V(aX + b) = a^2 V(X),$$

where 'a' and 'b' are constants. 1½

(e) Find moment generating function of a geometric distribution. 2

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Master of Science Mathematics 1st Sem. Examination – December, 2024

MATHEMATICAL STATISTICS

Paper : 24MAT201DS04

Time : Three Hours] *[Maximum Marks : 70*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt ***five*** questions in all, selecting ***one*** question from each Section. Question No. 1 is ***compulsory***. All questions carry equal marks.

1. (a) Give statistical definition of probability. 1½

(b) What are 'pairwise' and 'mutually' independent events ? 2

(c) Define a random variable. Give an example. 2

(d) For a random variable X, show that

$$V(aX + b) = a^2\, V(X),$$

where 'a' and 'b' are constants. 1½

(e) Find moment generating function of a geometric distribution. 2

(f) State under what conditions binomial and Poisson distributions tends to normal distribution. 1½

(g) Explain the term 'critical region' in testing of hypothesis. 1½

(h) Differentiate between 'Simple' and 'Composite' hypotheses. 2

SECTION – I

2. (a) What is axianatic definition of probability ? For two events A and B prove that for $B \subseteq A$

$$P(B) \leq P(A).$$ 4

(b) State and prove multiplication law of probability. 6

(c) A bag contains 6 white and 9 black balls. This drawings of 4 balls each are made from the bag such that the balls are not replaced before the second trial. What is the probability that the first drawing will give 4 black balls and second 4 white balls ? 4

3. (a) Define conditional probability. Let A, B and C be any three events such that B and C are independent, show that 6

$$P(A/B) = P(A/B \cap C) \,.\, P(C) + P(A/B \cap \bar{C}) \,.\, P(\bar{C})$$

(b) The odds that a book on statistics will be favourably reviewed by 3 independent critics are 3 to 2, 4 to 3 and 2 to 3 respectively. Then, obtain the probability that of three reviews atleast one of the review will be fovourable. 5

(c) What is the Baye's theorem in probability ? Give its uses. 3

SECTION – II

4. (a) Explain the following terms : 6
   (i) Probability density function
   (ii) Joint probability mass function
   (iii) Marginal probability density function, and
   (iv) Conditional distribution function

   (b) A two-dimensional random variable (X, Y) has a joint p.m.f. :

$$p(x, y) = \frac{1}{27}(x + 2y),$$

   for x and y can assume only the integer values 0, 1, and 2.

   Find (i) Marginal distribution of X and Y, and (ii) Conditional distribution of Y for $X = x$. 8

5. (a) What do you mean by mathematical expectation of a random variable ? For two discrete random variables, prove that

$$E(X + Y) = E(X) + E(Y).$$ 6

   (b) Let X be a random variable taking values 1, 2, 3 and 4 with probability 0.1, 0.2, 0.3 and 0.4, respectively. Find $V(2X + 3)$. 4

   (c) Write a note on moment generating function and its properties. 4

SECTION – III

6. (a) Define the binomial distribution. Obtain its mean and variance. Show that mean $\geq$ variance. 8

   (b) State and prove 'lack of memory property' of the exponential distribution. 6

7. (a) If X is uniformly distributed with mean 1 and variance 4/3' then find $P(X > 0)$ and $P(X < 2)$. 5

(b) What do you mean by a normal distribution ? State chief characteristics of the normal distribution and the normal curve. 9

SECTION – IV

8. (a) Distinguish between the following : 9

   (i) Parameter and statistics

   (ii) Null and alternative hypotheses

   (iii) One-tailed and two-tailed tests.

   (b) What is standard error of an estimate ? Explain, giving a suitable example. Also, give its utility. 5

9. (a) Explain test of significance of 'difference of two proportion' for large samples. 5

   (b) In a random sample of 500, the mean is found to be 20. In another independent sample of 400, the mean is 15. Test whether the samples have been drawn from the same population with standard deviation 4. [Given that $z_{.05} = 1.96$]. 6

   (c) In testing of hypothesis, what are two kinds of errors ? Explain. 3

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86095

Master of Science Mathematics

1st Semester

Examination – December, 2024

ANALYTICAL NUMBER THEORY

Paper : 24MAT201DS05

Time : Three Hours] *[Maximum Marks : 70*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt ***five*** questions in all, selecting ***one*** question from each Section. Question No. **1** is ***compulsory***. All questions carry equal marks.

1. (a) Prove that primes of the form $4n + 3$ are infinite in number. 14

(b) Prove that for $n \geq 2$, $10 / F_n - 7$.

(c) Show that 3 is a generator of U_7.

(d) Define quadratic residues.

(e) Define Euler's product.

(f) Show that g(2) = 4.

(g) Define perfect numbers.

SECTION – I

2. (a) Prove that all Fermat numbers are relatively prime to each other. 7

(b) Show that π is irrational. 7

3. (a) State and prove Hurwitz theorem. 7

(b) Let $\frac{h}{k}$ and $\frac{h'}{k'}$ be two successive members of F_n such that $\frac{h}{k} < \frac{h'}{k'}$, then prove that h'k – hk' = 1. 7

SECTION – II

4. (a) Let p be an odd prime. Let d divides $p^n(p-1)$ for $n \geq 1$. Prove that the group U_d is cyclic. 7

(b) Prove that group U_2n is cyclic if and only if n = 1 or n = 2. 7

5. (a) Prove that an element $a \in U_n$ is a primitive root if and only if $a^{\frac{\phi(n)}{q}} \neq 1$ in U_n for each prime q dividing $\phi(n)$. 7

(b) If p is a prime then the group U_p has $\phi(d)$ elements of order d for each d dividing p – 1. 7

SECTION – III

6. (a) Let p_n denotes the nth prime (in increasing order), then prove that the infinite series, $\sum_{n=1}^{\infty} \frac{1}{p_n} = \frac{1}{2} + \frac{1}{3} + \frac{1}{5} + \ldots\ldots\ldots$ is divergent. 7

(b) Prove that if x, y, z satisfies $x^2 + y^2 = z^2$, then 7

(i) $xyz \equiv 0 (\text{mod } 60)$

(ii) $xy(x^2 - y^2) \equiv 0 (\text{mod } 84)$

7. (a) Prove that $x^4 + y^4 = u^2$ has no non-trivial solution. 7

(b) Prove that $G(2^\theta) \geq 2^{\theta+2}$ for $\theta \geq 2$. 7

SECTION – IV

8. (a) Let n be any natural number then

$$\sum_{d/n} \mu(d) = \begin{cases} 1 & if \quad n = 1 \\ 0 & if \quad n > 1 \end{cases}$$ 7

(b) Prove that Euler's ϕ function, $\phi(n)$ is a multiplicative function. 7

9. (a) Prove that $\sigma(24m - 1) \equiv 0 (\text{mod } 24) \forall m \geq 1$. 7

(b) Prove that $d(n) = O(n^\delta)$ for all positive δ. 7

Roll No. ....

86097

Master of Science (Mathematics)
1st Semester
Examination – December, 2024

DISCRETE MATHEMATICS

Paper : 24MAT201SE01

Time : Three Hours] *[Maximum Marks : 70*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt *five* questions in all, selecting *one* question from each Section. Question No. 9 (Section - V) is *compulsory*. All questions carry equal marks.

SECTION – I

1. (a) Find an explicit formula for the recurrence relation $a_0 = 1, a_n = a_{n-1} + 2$. 7

(b) Find an explicit formula for the sequence defined by the recurrence relation $a_n = a_{n-1} + 2a_{n-2}, n \geq 2$ with the initial conditions $a_0 = 1$ and $a_1 = 8$. 7

2. (a) Find the particular solution of the difference equation $a_n - 5a_{n-1} + 4a_{n-2} = 56.\, 3^n$. Hence find the total solution of this difference equation. 7

(b) Using generating function methods, find the explicit formula for Fibonacci sequence. 7

SECTION – II

3. (a) Define propositional function by giving two examples. If $P(x)$: x is even ; $Q(x)$: x is a prime number ; $R(x, y)$: $x + y$ is even. The variables x and y represent integers, then write each of the following in terms of $P(x)$, $Q(x)$, $R(x, y)$, logical connectives and quantifiers : 7

(i) Every integer is an odd integer.

(ii) The sum of any two integers is an even number.

(iii) There are no even prime numbers.

(iv) Every integer is even or a prime.

(b) A software engineer makes the following observations in a computer programming : 7

(i) There is an undeclared variable or there is syntax error in the first five lines.

(ii) If there is a syntax error in the first five lines, then there is a missing semicolon or a variable name is misspelled.

(iii) There is not a missing semicolon.

(iv) There is not a misspelled variable name. Using logical forms, find the mistake in the program.

4. (a) Let L be a complemented lattice with unique complements. Then show that the join irreducible elements of L, other than 0, are its atoms. 7

(b) Show that the idempotent laws follow from the absorption laws. 7

SECTION – III

5. (a) Let $n = p_1 p_2 .....p_k$ where p_k where p_i are distinct primes, known as set of atoms. Show that D_n is a Boolean algebra. 7

(b) Find the prime implicant of $E = xyz + x'z' + xyz' + x'y'z + x'yz'$. 7

6. (a) Show that for any two elements a and b in a Boolean algebra : 7

(i) $(a \vee b)' = a' \wedge b'$

(ii) $(a \wedge b)' = a' \vee b'$

(b) Let B = {1, 2, 5, 7, 10, 14, 35, 70}. For any a, b in B, define +, * and as follows :

$a + b = lcm(a, b)$,

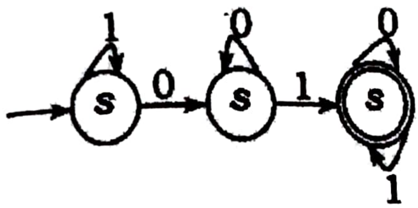
$a * b = \gcd(a, b), a' = \frac{70}{a}$

Then show that B is a Boolean algebra with 1 as zero element and 70 as unit element. 7

SECTION – IV

7. (a) Show that language $L = \{a^p : p \text{ is prime}\}$ is not regular. 7

(b) A finite state automaton M is described by the following diagram :



Find the corresponding grammar G and L(G). 7

8. (a) Let L be a set accepted by a non-deterministic finite automaton. Then prove that there exists a deterministic finite automaton that accepts L. 7

(b) Let M_1 be a mealy Machine whose transition table is given by :

	f		G	
S\I	0	1	0	1
s_0	s_2	s_1	0	0
s_1	s_0	s_3	1	0
s_2	s_1	s_0	1	1
s_3	s_3	s_2	1	0

Find equivalent Moore machine M_2. 7

SECTION – V

9. (a) Define linear recurrence relation of order k. 14

(b) Define generating function.

(c) Define Boolean function.

(d) Define AND Gate.

(e) Discuss the contradiction rule.

(f) Define complete lattice with example.

(g) Define Equivalence of Finite State Machines.

(h) Define Ambiguous Grammar.